Table of Contents

About Singapore Math vii
Before You Begin ix
Components of a Lesson xi

About the Course

Course Materials xiii
Course Survey xiv
Types of Lessons xv
Constructed Responses xvi

The Learning Team

The Role of the Student xviii
The Role of the Learning Guide xviii
The Role of the Calvert Support Staff xix

Testing

Calvert Test Series xx
Advisory Teaching Service Tests xx
Directions for Administering Tests xx
Submitting the Tests xxi
Test Submission Schedule xxi
Express ATS xxii
Certificate of Completion xxiii

Frequently Asked Questions

What if I have a question about the curriculum? xxiv
How do I contact Calvert by mail? xxiv
How do I send my first Advisory Teaching Service Test? xxiv
When is my student eligible to receive a Certificate of Completion? xxiv
What should I do if I find a mistake in the manual? xxv
How can I get a transcript of my student's grades? xxv

Lessons

Lesson 1: Chapter 1 Opener
Positive Numbers on the Number Line 1

Lesson 2
Exploring the Number Line 3

Lesson 3
Comparing Numbers 5

Lesson 4
Finding the Prime Factorization 7

Lesson 5
Finding Common Factors 9

Lesson 6
Finding Common Multiples 11

Lesson 7
Finding Squares and Square Roots 13

Lesson 8
Finding Cubes and Cube Roots 15

Lesson 9
Evaluating Expressions with Exponents 17

Lesson 10
Using Bar Models to Solve Problems 19

Lesson 11
More Bar Models 21

Lesson 12
Bar Models with Multiplication 23

Lesson 13
Problem Solving 25

Lesson 14
Chapter Review 27

Chapter 1 Test 29

Lesson 15: Chapter 2 Opener
Negative Numbers on the Number Line 35

Lesson 16
Exploring Negative Numbers 37

Lesson 17
More Comparing Numbers 39

Lesson 18
Finding Absolute Value 41

Lesson 19
Problem Solving 43

Lesson 20
Chapter Review 45

Chapter 2 Test 47

Lesson 21: Chapter 3 Opener
Multiplication and Division with Fractions and Decimals 53

Lesson 22
Adding Fractions 55

Lesson 23
Subtracting Fractions 57

Lesson 24
Subtracting Whole and Mixed Numbers 59

Lesson 25
Bar Models with Division 61

Lesson 26
Bar Models with Fractions 63

Lesson 27
Dividing Whole Numbers by Unit Fractions 65

Lesson 28
Dividing Whole Numbers by Fractions 67

Lesson 29
Dividing Fractions by Fractions 69

Lesson 30
Dividing Fractions by Improper Fractions and Mixed Numbers 71

Lesson 31
Multiplication with Decimals 73

Lesson 32
More Multiplication with Decimals 75

Lesson 33
Division with Decimals 77

Lesson 34
More Division with Decimals 79

Lesson 35
Solving Problems with Decimals and Fractions 81

Lesson 36
More Solving Problems with Decimals and Fractions 83

Lesson 37
Practice Solving Problems with Decimals and Fractions 85

Lesson 38
Problem Solving 87

Lesson 39
Chapter Review 89

Chapter 3 Test 91

Lesson 40: Chapter 4 Opener
Exploring Ratio 97

Lesson 41
Measurements and Comparison Bar Models 99

Lesson 42
The Meaning of Ratios 101

Lesson 43
More about Ratios 103

Lesson 44
Finding Equivalent Ratios 105

Lesson 45
Finding Missing Terms in Equivalent Ratios 107

Lesson 46
Solving Problems with Ratios 109

Lesson 47
More Solving Problems with Ratios 111

Lesson 48
Problem Solving 113

Lesson 49
Chapter Review 114

Chapter 4 Test 115

Lesson 50: Chapter 5 Opener
Exploring Rates 121

Lesson 51
Identifying Rates and Unit Rates 123

Lesson 52
Comparing Unit Rates 125

Lesson 53
Solving Real-World Problems with Rates 127

Lesson 54
More Real-World Problems with Rates 129

Lesson 55
Measurement Problems 131

Lesson 56
Problem Solving 133

Lesson 57
Chapter Review 135

Chapter 5 Test 137

Lesson 58: Chapter 6 Opener
Exploring Percents 143

Lesson 59
The Meaning of Percent 145

Lesson 60
Relating Fractions, Decimals, and Percents 147

Lesson 61
Finding Percent of a Whole 149

Lesson 62
Solving Real-World Percent Problems 151

Lesson 63
Using the Part and the Percent to Find the Whole 153

Lesson 64
Finding Percent of Change 155

Lesson 65
Real-World Problems with Percent of Change 157

Lesson 66
Problem Solving 159

Lesson 67
Chapter Review 161

Chapter 6 Test 163

Lesson 68: Chapter 7 Opener
Exploring Algebraic Expressions 167

Lesson 69
Writing an Algebraic Expression for Addition and Subtraction 169

Lesson 70
Writing an Algebraic Expression for Multiplication and Division 171

Lesson 71
Evaluating an Algebraic Expression 173

Lesson 72
Simplifying an Algebraic Expression 175

Lesson 73
Adding Like Terms 177

Lesson 74
Using the Order of Operations to Simplify Expressions 179

Lesson 75
Using the Distributive Property with Expressions 181

Lesson 76
Finding the Common Factor in an Algebraic Expression 183

Lesson 77
Solving Real-World Problems with Expressions 185

Lesson 78
More Solving Real-World Problems with Expressions 187

Lesson 79
Problem Solving 189

Lesson 80
Chapter Review 191

Chapter 7 Test 193

Appendix

General Resources
Grid Paper 201
Fraction Strips 205
Counters 207

Worksheets and Projects
Using Bar Models to Solve Problems Worksheet (Lesson 10) 213
More Bar Models Worksheet (Lesson 11) 217
Bar Models with Multiplication Worksheet (Lesson 12) 222
Adding Fractions Worksheet (Lesson 22) 228
Subtracting Fractions Worksheet (Lesson 23) 233
Subtracting Whole and Mixed Numbers Worksheet (Lesson 24) 237
Bar Models with Division Worksheet (Lesson 25) 241
Bar Models with Fractions Worksheet (Lesson 26) 245
Solving Problems with Decimals and Fractions Worksheet (Lesson 37) 250
Measurement Problems Worksheet (Lesson 55) 259
Make Your Own Currency Project (Lesson 56) 265

Online Resources 269

Table of Contents

Before You Begin vii

Lesson 81: Chapter 8 Opener
Exploring Equations and Inequalities 275

Lesson 82
Solving Equations by Substitution 277

Lesson 83
Solving Equations with Addition and Subtraction 279

Lesson 84
Solving Equations with Multiplication and Division 281

Lesson 85
Solving Equations with Fractions 283

Lesson 86
Practice with Solving Equations 285

Lesson 87
Writing a Linear Equation 287

Lesson 88
Using Tables and Graphs to Write Linear Equations 289

Lesson 89
Inequalities 291

Lesson 90
More Inequalities 293

Lesson 91
Solving Problems with Equations 295

Lesson 92
Solving Problems with Inequalities 297

Lesson 93
Problem Solving 299

Lesson 94
Chapter Review 301

Chapter 8 Test 303

Lesson 95: Chapter 9 Opener
Graphing on the Coordinate Plane 309

Lesson 96
Finding Points on the Coordinate Plane 311

Lesson 97
Polygons on the Coordinate Plane 313

Lesson 98
Finding the Length of Line Segments Parallel to the Axes 315

Lesson 99
Plotting Points on a Coordinate Plane to Solve Problems 317

Lesson 100
Using Tables and Graphs to Solve Real-World Problems 319

Lesson 101
Problem Solving 321

Lesson 102
Chapter Review 323

Chapter 9 Test 325

Lesson 103: Chapter 10 Opener
Finding the Area of Polygons 333

Lesson 104
Finding the Area of a Triangle Using a Formula 335

Lesson 105
Solving Problems Using the Formula for Area of a Triangle 337

Lesson 106
Finding the Area of Parallelograms 339

Lesson 107
Finding the Area of Trapezoids by Using a Formula 341

Lesson 108
Solving Multistep Problems Using Area Formulas 343

Lesson 109
Finding the Area of Regular Polygons.....345

Lesson 110
Finding the Area of Other Polygons 347

Lesson 111
More Area of Other Polygons 349

Lesson 112
Problem Solving 351

Lesson 113
Chapter Review.....353

Chapter 10 Test.....355

Lesson 114: Chapter 11 Opener
Circles 361

Lesson 115
Parts of a Circle 363

Lesson 116
Finding Circumference 365

Lesson 117
Finding the Area of a Circle.....367

Lesson 118
Finding the Area of Semicircles and Quadrants.....369

Lesson 119
Solving Problems with Area and Circumference.....371

Lesson 120
Solving Problems with Rates and Circles.....373

Lesson 121
Problem Solving.....375

Lesson 122
Chapter Review.....377

Chapter 11 Test.....379

Lesson 123: Chapter 12 Opener
Surface Area and Volume 385

Lesson 124
Nets of Prisms.....387

Lesson 125
Nets of Pyramids.....389

Lesson 126
Surface Area of Cubes and Rectangular Prisms 391

Lesson 127
Surface Area of Triangular Prisms and Pyramids.....393

Lesson 128
Finding the Volume of a Prism.....395

Lesson 129
More About Prisms.....397

Lesson 130
Surface Area and Volume Problems 399

Lesson 131
More Surface Area and Volume Problems.....401

Lesson 132
Problem Solving 403

Lesson 133
Chapter Review.....405

Chapter 12 Test.....407

Lesson 134: Chapter 13 Opener
Statistics 413

Lesson 135
Statistical Questions.....415

Lesson 136
Gathering Data.....417

Lesson 137
Working with Dot Plots.....419

Lesson 138
More Dot Plots 421

Lesson 139
Creating Bar Graphs 423

Lesson 140
Interpreting a Bar Graph 425

Lesson 141
Creating Histograms 427

Lesson 142
More Work with Histograms 429

Lesson 143
Interpreting Histograms.....431

Lesson 144
Bar Graph or Histogram? 433

Lesson 145
Problem Solving.....435

Lesson 146
Chapter Review.....437

Chapter 13 Test.....439

Lesson 147: Chapter 14 Opener
Measures of Center.....447

Lesson 148
Finding the Mean.....449

Lesson 149
More About Means.....451

Lesson 150
Finding the Median 453

Lesson 151
More About Medians 455

Lesson 152
Finding the Mode 457

Lesson 153
Modes and Nonnumerical Sets 459

Lesson 154
Comparing Measures of a Data Set.....461

Lesson 155
Choosing a Measure of Center.....463

Lesson 156
Measures of Central Tendency and Distribution.....465

Lesson 157
Interpreting Box-and-Whisker Plots.....467

Lesson 158
Creating Box-and-Whisker Plots 469

Lesson 159
Your Statistics Project.....471

Lesson 160
Chapter Review.....473

Chapter 14 Test..... 475

General Resources
Grid Paper 485
Pattern Blocks 545
Algebra Tiles 547
Prism and Pyramid Nets 549

Worksheets and Projects
Single-Step Equations Worksheet (Lesson 86) 559
Statistical Questions Worksheet (Lesson 135).....562
Your Statistics Project (Lesson 135) 564
Creating Bar Graphs Worksheet (Lesson 139).....569
Interpreting a Bar Graph Worksheet (Lesson 140).....574
Working with Histograms Worksheet (Lesson 142).....579
Bar Graph vs. Histogram Worksheet (Lesson 144).....584
Measures of a Data Set Worksheet (Lesson 154).....587
Interpreting Box-and-Whisker Plots Worksheet (Lesson 157).....590
Creating Box-and-Whisker Plots Worksheet (Lesson 158).....593

Online Resources 597

Multiplication and Division with Fractions and Decimals

CCSS 6.NS.3
K–12.MP2

Objectives
- Add and subtract decimals.
- Rename improper fractions as mixed numbers and mixed numbers as improper fractions.
- Multiply fractions by fractions.

Books & Materials
- *Math in Focus* A

Assignments
- ❑ Complete Warm-up.
- ❑ Read and complete pp. 62–64, *Math in Focus* A.
- ❑ Complete Math Checkpoint.

Warm-up

Add or subtract.

1. 68 + 54
2. 475 − 227

Instruction

Today's lesson covers important ideas that you need to understand before beginning the lessons in this chapter. Read p. 62 in ***Math in Focus.*** Then read **Recall Prior Knowledge** on pp. 63–64.

Adding and subtracting decimals is similar to adding and subtracting whole numbers. When you work with decimals, however, you must remember to place the decimal point in the sum or difference. Sometimes, before you can add or subtract, you need to add zeros so that the numbers in a problem have the same number of decimal places.

In the second instructional section on p. 63, practice renaming an improper fraction as a mixed number by dividing the numerator of the improper fraction by the denominator. The number of times the denominator goes into the numerator is the whole-number part of the mixed number, and the remainder is the numerator of the fraction part. The denominator of the fraction part remains the same.

In the first instructional section on p. 64, reverse the process to rename a mixed number as an improper fraction. Multiply the whole-number part by the denominator and then add the numerator to the product. The denominator remains the same.

In the second instructional section on p. 64, recall that there are two approaches to multiplying a fraction by a fraction. In Method 1, multiply across numerators and across denominators and then simplify the product. When numerators and denominators share common factors, you can use Method 2. Divide a numerator and a denominator by a common factor first. Then multiply across the numerators and denominators. You will not need to simplify the final fraction using this method unless you need to rename an improper fraction as a mixed number.

Helpful Online Resources

- **Instructional Video:** *Adding and Subtracting Decimals*
- **Instructional Video:** *Writing Mixed Numbers as Improper Fractions*
- **Instructional Video:** *Multiplying Two Fractions*

To the Learning Guide

As you discuss these pages with your student, take note of the skills that seem familiar and those that may need review. Do not provide detailed instruction at this point; simply preview the material to prepare for the **Quick Check** sections.

Skills Check

Complete the **Quick Check** sections on pp. 63–64 in ***Math in Focus***.

To the Learning Guide

Review your student's answers to the **Quick Check** sections, noting the problems that he answered incorrectly. Then go online through your student's portal to today's lesson. Click on the link to access the appropriate **Reteach** activity that your student should complete for the remainder of this lesson.

Wrap-up

Today you reviewed how to add and subtract decimals with and without regrouping.

$$\begin{array}{r} {\scriptstyle 1} \\ 6.38 \\ +\ 5.45 \\ \hline 11.83 \end{array} \qquad \begin{array}{r} {\scriptstyle 6\,11\,14} \\ 7.\cancel{2}\cancel{4} \\ -\ 4.67 \\ \hline 2.57 \end{array}$$

You reviewed how to rename an improper fraction as a mixed number and how to rename a mixed number as an improper fraction.

$$\frac{17}{4} = \frac{16}{4} + \frac{1}{4} \qquad 3\frac{4}{5} = 3 + \frac{4}{5}$$
$$= 4 + \frac{1}{4} \qquad = \frac{15}{5} + \frac{4}{5}$$
$$= 4\frac{1}{4} \qquad = \frac{19}{5}$$

You also reviewed how to multiply fractions in two ways. For easier computation and less simplifying at the end, divide numerators and denominators by common factors before multiplying.

$$\frac{3}{4} \times \frac{5}{12} = \frac{3 \div 3}{4} \times \frac{5}{12 \div 3}$$
$$= \frac{1 \times 5}{4 \times 4}$$
$$= \frac{5}{16}$$

☑ **Complete Math Checkpoint**

06M-LMA

CHAPTER

3

Multiplying and Dividing Fractions and Decimals

3.1 Dividing Fractions

3.2 Multiplying Decimals

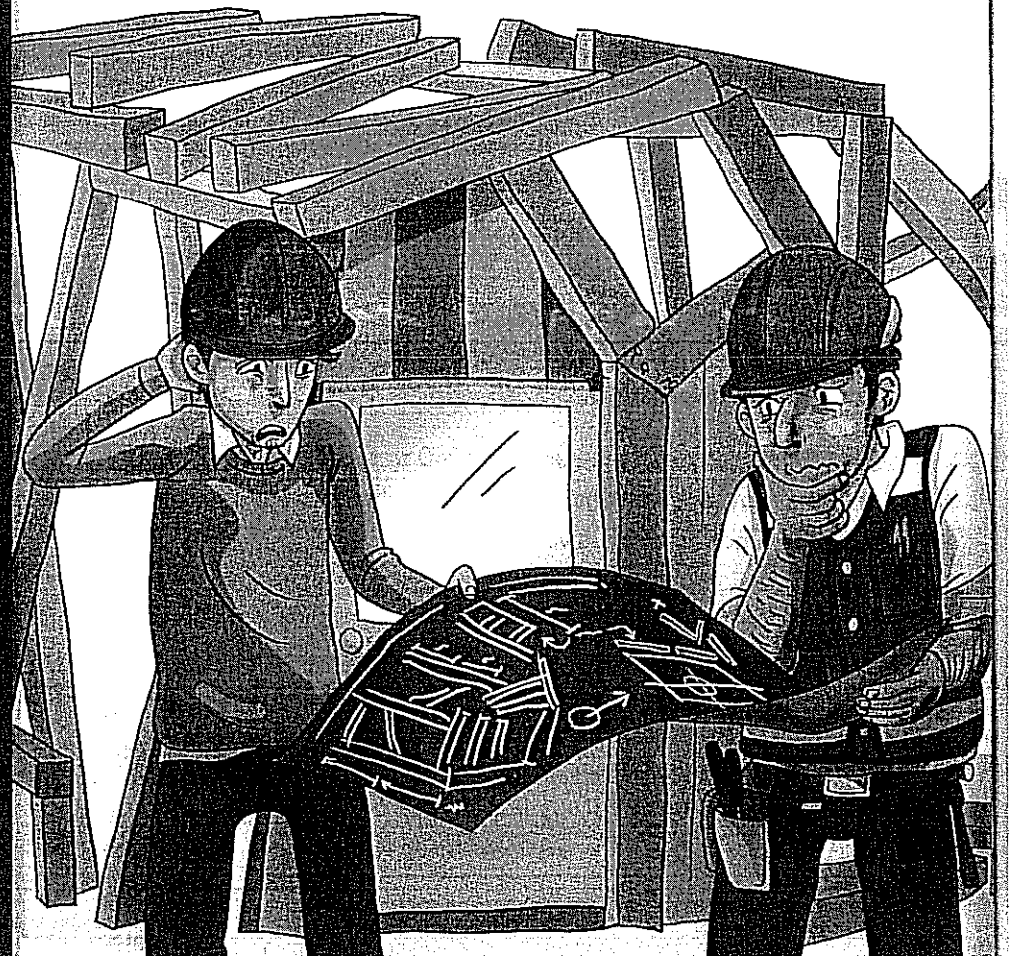
3.3 Dividing Decimals

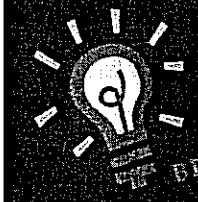
3.4 Real-World Problems: Fractions and Decimals

How much do you know about the construction of your home?

Design and cost are important factors for home builders. Architects have to manage the use of construction materials for the floors, walls, doors, windows, electrical fittings, and more. For example, to build a wooden-framed house, architects need to determine how many joists to use if the joists are $1\frac{1}{3}$ feet apart.

Architects must also calculate the costs of materials and labor. These calculations involve fractions and decimals. In this chapter, you will practice multiplying and dividing with fractions and decimals. The skills you learn will be useful for solving many real-world problems.





▶ Whole number concepts can be extended to fractions and decimals when more precise calculations are needed.

Recall Prior Knowledge

Adding and subtracting decimals

$$\begin{array}{r} 3.8 \\ +\ 2.1 \\ \hline 5.9 \end{array} \qquad \begin{array}{r} {\scriptstyle 1\ \ 1} \\ 7.86 \\ +\ 4.75 \\ \hline 12.61 \end{array}$$

$$\begin{array}{r} 8.26 \\ -\ 7.03 \\ \hline 1.23 \end{array} \qquad \begin{array}{r} {\scriptstyle 5\ \ 10\ 1} \\ \not{6}.\not{1}0 \\ -\ 2.34 \\ \hline 3.76 \end{array}$$

Quick Check

Add or subtract.

1. 5.3 + 6.49
2. 6.51 − 2.03
3. 9.62 + 7.08
4. 8.4 − 7.52

Expressing improper fractions as mixed numbers

$\frac{16}{5} = \frac{15}{5} + \frac{1}{5}$ Rewrite as a sum.

$= 3 + \frac{1}{5}$ Write the improper fraction as a whole number.

$= 3\frac{1}{5}$ Write the sum as a mixed number.

Quick Check

Express each improper fraction as a mixed number in simplest form.

5. $\frac{19}{3}$
6. $\frac{26}{4}$
7. $\frac{30}{7}$
8. $\frac{38}{5}$
9. $\frac{50}{8}$
10. $\frac{69}{9}$

Expressing mixed numbers as improper fractions

$2\frac{5}{6} = 2 + \frac{5}{6}$ Rewrite as a sum.

$= \frac{12}{6} + \frac{5}{6}$ Write the whole number as a fraction.

$= \frac{17}{6}$ Write the sum as an improper fraction.

Quick Check

Express each mixed number as an improper fraction.

11 $3\frac{1}{4}$ 12 $4\frac{3}{7}$ 13 $8\frac{5}{9}$

Multiplying fractions by fractions

Method 1

$\frac{3}{4} \times \frac{2}{9} = \frac{3 \times 2}{4 \times 9}$ Multiply the numerators. Multiply the denominators.

$= \frac{6}{36}$ Simplify the product.

$= \frac{1}{6}$ Write the fraction in simplest form.

Method 2

$\frac{3}{4} \times \frac{2}{9} = \frac{3 \div 3}{4} \times \frac{2}{9 \div 3}$ Divide a numerator and a denominator by the common factor, 3.

$= \frac{1}{4 \div 2} \times \frac{2 \div 2}{3}$ Divide a numerator and a denominator by the common factor, 2.

$= \frac{1 \times 1}{2 \times 3}$ Multiply the numerators. Multiply the denominators.

$= \frac{1}{6}$

Quick Check

Find each product in simplest form.

14 $\frac{2}{5} \times \frac{7}{8}$ 15 $\frac{10}{11} \times \frac{33}{5}$ 16 $\frac{8}{7} \times \frac{35}{12}$

3.1 Dividing Fractions

Lesson Objective

- Divide a fraction, whole number, or mixed number by a fraction or a mixed number.

Vocabulary

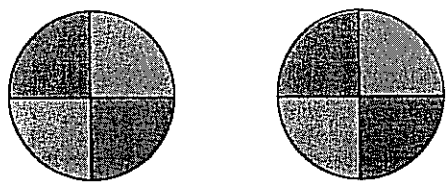
reciprocals
mixed number
improper fraction

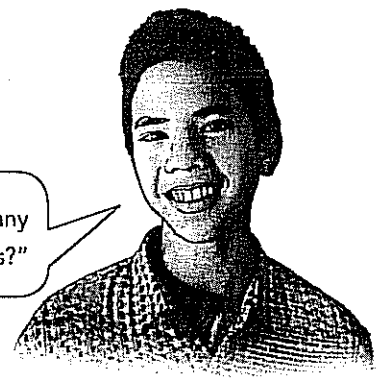
Learn

Divide a whole number by a unit fraction.

a) Lee cut 2 pies into pieces that were each $\frac{1}{4}$ of a pie.
How many pieces did Lee cut the 2 pies into?

Number of pieces $= 2 \div \frac{1}{4}$



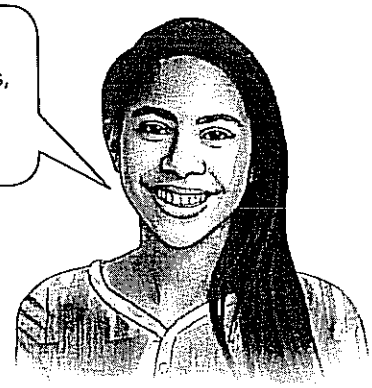


$2 \div \frac{1}{4}$ means: "How many quarters are in 2 wholes?"

The model shows that:
Number of quarters in 1 pie $= 4$
Number of quarters in 2 pies $= 2 \times 4$

So, $2 \div \frac{1}{4} = 2 \times 4$ Write as a multiplication expression.
$= 8$ Multiply.

Lee cut the 2 pies into 8 pieces.



In one pie, there are 4 quarters. So in two pies, there are $2 \times 4 = 8$ quarters.

Math Note

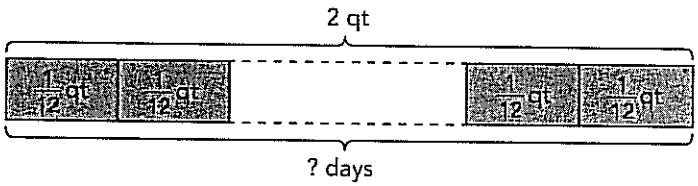
Two numbers whose product is 1 are called **reciprocals**. In general, dividing by a number is the same as multiplying by the reciprocal of the number.

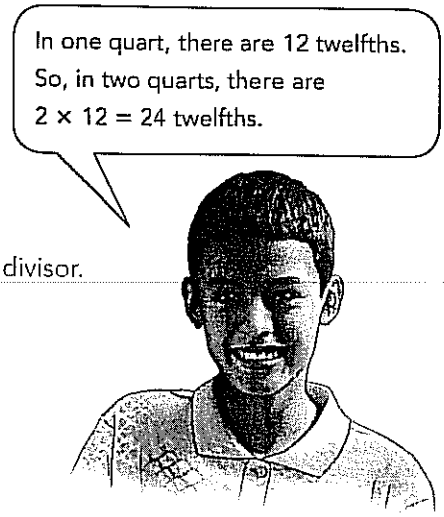
$2 \div \frac{1}{4} = 2 \times 4$

$\frac{1}{4}$ and **4** are reciprocals.

Continue on next page

b) A bottle contains 2 quarts of cooking oil. A chef uses $\frac{1}{12}$ quart of cooking oil each day. How many days will the bottle of cooking oil last?





$2 \div \frac{1}{12} = 2 \times 12$ Rewrite using the reciprocal of the divisor.

$= 24$ Multiply.

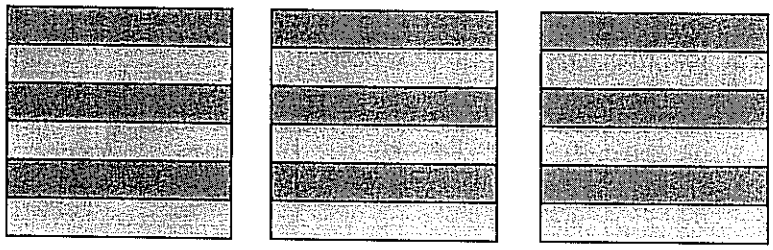
The bottle of cooking oil will last 24 days.

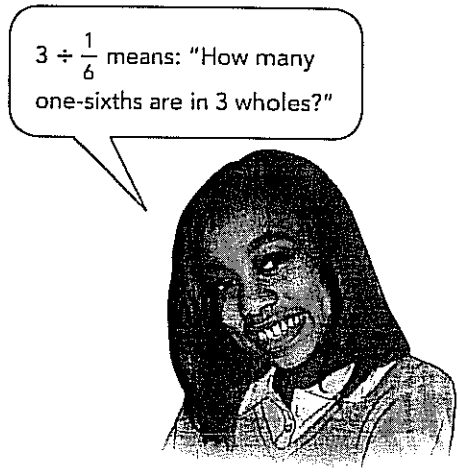
Guided Practice

Solve.

1 Rina cut 3 paper squares into a number of equal pieces. Each piece was $\frac{1}{6}$ of a square. Into how many pieces did Rina cut the 3 paper squares?

Number of pieces = $\frac{?}{?} \div \frac{?}{?}$





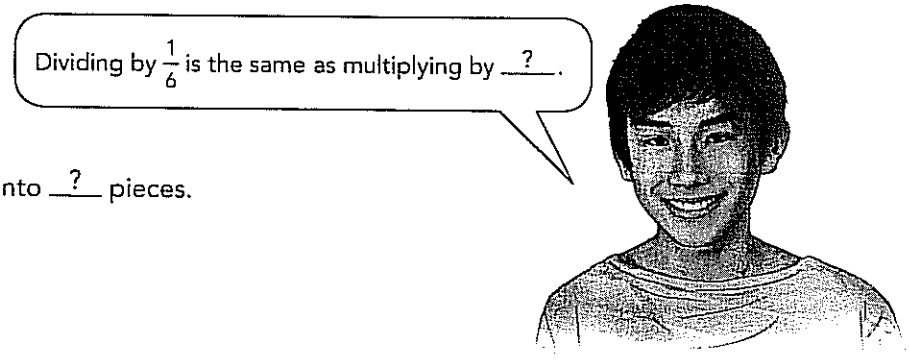
The model shows that:

Number of one-sixths in 1 paper square = ___?___

Number of one-sixths in 3 paper squares = ___?___ × ___?___

So, $3 \div \frac{1}{6} =$ ___?___ × ___?___

$=$ ___?___



Rina cut the 3 paper squares into ___?___ pieces.

Divide.

2. $3 \div \frac{1}{5} = \underline{?} \times \underline{?} = \underline{?}$

3. $7 \div \frac{1}{4} = \underline{?} \times \underline{?} = \underline{?}$

4. $4 \div \frac{1}{2} = \underline{?}$

5. $5 \div \frac{1}{3} = \underline{?}$

6. $6 \div \frac{1}{5} = \underline{?}$

7. $8 \div \frac{1}{8} = \underline{?}$

Hands-On Activity

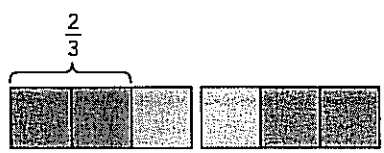
Materials:

- 5 paper strips

DIVIDING WHOLE NUMBERS BY A FRACTION

Use 5 paper strips of the same size and length. Each strip represents 1 whole.

STEP 1 Take 2 paper strips. Divide each of them into thirds using vertical lines and place them as shown. Then find $2 \div \frac{2}{3}$.



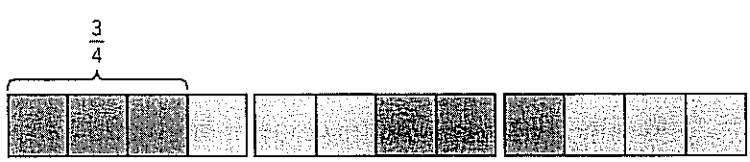
Refer to your model.

Complete:

There are $\underline{?}$ two-thirds in the 2 paper strips.

So, $2 \div \frac{2}{3} = \underline{?}$.

STEP 2 Divide each of the other 3 paper strips into fourths using vertical lines and place them as shown. Then find $3 \div \frac{3}{4}$.



Refer to your model.

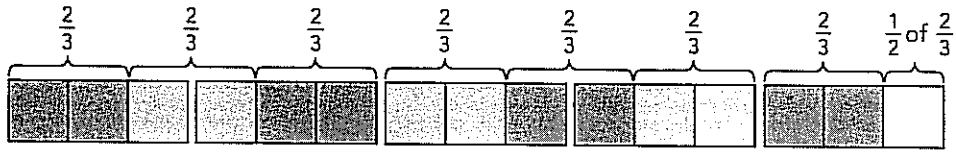
Complete:

There are $\underline{?}$ three-fourths in the 3 paper strips.

So, $3 \div \frac{3}{4} = \underline{?}$.

Learn — Divide a whole number by a proper fraction.

a) Find $5 \div \frac{2}{3}$.



Number of two-thirds in 2 wholes = 3

Number of two-thirds in 1 whole = $1\frac{1}{2}$ or $\frac{3}{2}$

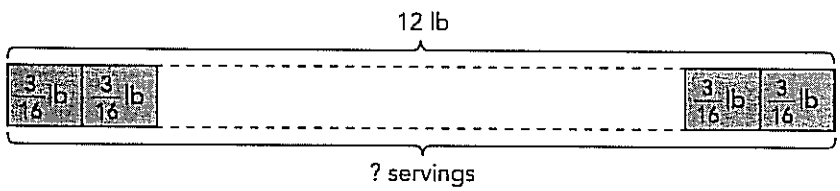
Number of two-thirds in 5 wholes = $5 \times \frac{3}{2}$

So, $5 \div \frac{2}{3} = 5 \times \frac{3}{2}$

$= \frac{15}{2}$

$= 7\frac{1}{2}$

b) A chef cooks 12 pounds of pasta each day. She uses $\frac{3}{16}$ pound of pasta for each serving she prepares. How many servings of pasta does she prepare each day?



$12 \div \frac{3}{16} = 12 \times \frac{16}{3}$ Rewrite using the reciprocal of the divisor.

$= 64$ Multiply.

She prepares 64 servings of pasta each day.

Check: Calculate the total number of pounds of pasta she cooks.

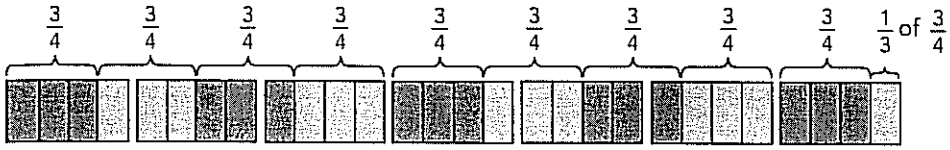
$64 \times \frac{3}{16} = 12$

The answer is correct.

Guided Practice

Complete.

8 Find $7 \div \frac{3}{4}$.



Number of three-quarters in 3 wholes = 4

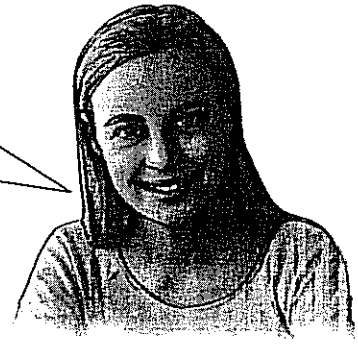
Number of three-quarters in 1 whole = $1\frac{1}{3}$ or $\frac{?}{?}$

Number of three-quarters in 7 wholes = $\frac{?}{\quad}$ × $\frac{?}{?}$

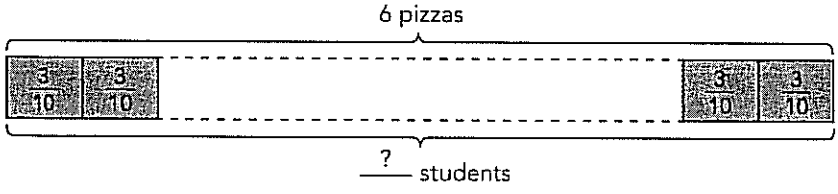
So, $7 \div \frac{3}{4} = \frac{?}{\quad} \times \frac{?}{?}$

$= \frac{?}{?}$

$= \frac{?}{\quad}$



9 Mrs. Johnson bought 6 pizzas. She cut them into many equal pieces for the students in her class. Each piece was $\frac{3}{10}$ of a whole pizza. How many students were there in the class if each child received only one piece of pizza?



$6 \div \frac{?}{?} = \frac{?}{\quad} \times \frac{?}{?}$

$= \frac{?}{\quad}$

There were ___ students in the class.

Divide. Express the quotient in simplest form.

10 $4 \div \frac{4}{7}$

11 $6 \div \frac{2}{7}$

12 $9 \div \frac{3}{8}$

13 $5 \div \frac{10}{13}$

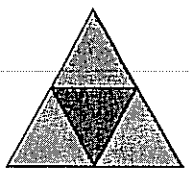
14 $10 \div \frac{5}{14}$

15 $12 \div \frac{9}{10}$

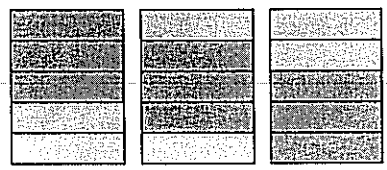
Practice 3.1

Divide. Express the quotient in simplest form. Use models to help you.

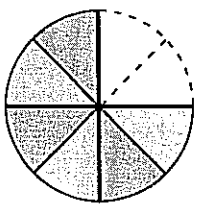
1. $1 \div \frac{1}{4}$



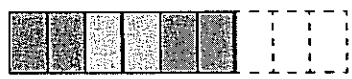
2. $3 \div \frac{3}{5}$



3. $\frac{3}{4} \div \frac{1}{8}$



4. $\frac{2}{3} \div \frac{2}{9}$



Draw a model to find each quotient.

5. $1 \div \frac{1}{5}$

6. $4 \div \frac{8}{9}$

7. $\frac{2}{5} \div \frac{3}{10}$

8. $\frac{3}{4} \div \frac{3}{16}$

Find each quotient. Express your answer in its simplest form.

9. $4 \div \frac{1}{7}$

10. $12 \div \frac{1}{3}$

11. $9 \div \frac{3}{4}$

12. $10 \div \frac{4}{5}$

13. $\frac{1}{2} \div \frac{1}{8}$

14. $\frac{1}{4} \div \frac{1}{2}$

15. $\frac{3}{5} \div \frac{11}{15}$

16. $\frac{2}{3} \div \frac{10}{13}$

17. $\frac{5}{6} \div \frac{7}{12}$

18. $\frac{3}{4} \div \frac{9}{16}$

Find each quotient. Express your answer in its simplest form.

19. $\frac{1}{3} \div \frac{7}{4}$

20. $\frac{1}{2} \div \frac{8}{3}$

21. $\frac{1}{9} \div \frac{14}{3}$

22. $\frac{5}{8} \div \frac{21}{4}$

23. $3\frac{1}{2} \div 2\frac{1}{8}$

24. $5\frac{1}{4} \div 3\frac{1}{2}$

25. $7\frac{3}{5} \div 8\frac{11}{15}$

26. $12\frac{2}{3} \div 5\frac{11}{13}$

Solve. Show your work.

27. 6 pizzas were shared equally among a group of children. Each child got $\frac{1}{9}$ of a pizza. How many children were in the group?

28. A rectangle has an area of 15 square meters. It is divided into parts, each with an area of $\frac{3}{8}$ square meter. Into how many parts has the rectangle been divided?

29. How many $\frac{3}{8}$-cup servings are in a pitcher containing $6\frac{3}{4}$ cups of orange juice?

30. Maria buys $8\frac{1}{3}$ pounds of beef to make tacos for a party. She uses $\frac{5}{9}$ pound of beef for each taco. How many tacos can Maria make?

31. A rectangular plot of land has an area of $\frac{1}{8}$ square mile. Its width is $\frac{3}{20}$ mile. What is the length of the plot of land?

32. A farmer has four plots of land, each with an area of 12 acres. He divides them into a number of parts, each with an area of $\frac{8}{9}$ acre. How many parts are there on the four plots of land?

33. The capacity of a large milk carton is $1\frac{1}{2}$ liters. A dozen large cartons are poured into a container and then poured into small cartons that each hold $\frac{3}{10}$ liter. How many small cartons of milk can be filled?

Lesson 77

Solving Real-World Problems with Expressions

CCSS 6.EE.2a, 2b, 2c, 3
K–12.MP1, 4

Objectives
- Solve real-world problems with algebraic expressions.

Books & Materials
- *Math in Focus* A

Assignments
- ❑ Complete Warm-up.
- ❑ Read and complete pp. 246–249, *Math in Focus* A.
- ❑ Complete problems 1–2, p. 251, *Math in Focus* A.
- ❑ Complete Math Checkpoint.

Warm-up

Write an expression to represent each situation.

1. Jillian has n stamps. She receives 3 more stamps. How many stamps does she have now?
2. If David has $n + 4$ stamps, and each stamp is worth 46 cents, what is the total value of David's stamps?
3. Billy has b stamps. If each stamp is worth 46 cents, what is the total value of Billy's stamps?

Instruction

Read **Learn** on p. 246 in ***Math in Focus***. In this example, you will learn to write algebraic expressions involving addition or subtraction to represent real-world problems. Then you will assign a value to the variable in an expression so you can evaluate the expression.

To solve real-world problems involving algebraic expressions, it helps to draw and label a bar model to represent each situation. As you draw the bar model, ask yourself: What information is given? What am I being asked to find?

Complete **Guided Practice** on p. 247. Keep in mind that variables represent actual numbers. Check your answers by comparing the ages you found for Kayla and Isaac against the facts in the original problem. Your answers should make the original statement true.

Read **Learn** on pp. 247–248. In this example, you will learn to write algebraic expressions involving multiplication or division to represent real-world problems. Then you will assign a value to the variable in an expression so you can evaluate the expression.

Complete **Guided Practice** on pp. 248–249. To solve this problem, find the unit rate to help you decide whether to multiply or divide.

To the Learning Guide

If your student struggles with **Guided Practice** problem 2, help him decide whether to multiply or divide. In part **a**, he should multiply to find the number of miles traveled. Point out that miles/gallon • gallons = miles. In part **b**, your student should divide to find the number of gallons used. Explain that miles ÷ miles/gallon = gallons.

Practice

Complete problems 1–2 of **Practice 7.5** on p. 251 in ***Math in Focus***.

To the Learning Guide

If your student has difficulty determining what information is given and what information is being asked for, help him deconstruct the problem one sentence at a time. Encourage your student to draw and label a bar model to help him solve each problem.

Remind your student of the importance of checking his work after solving problems involving variables. When a value for the variable is not given, he can substitute a hypothetical value for the variable. This strategy can help him determine whether or not his answers are correct.

Wrap-up

Today you learned how to solve real-world problems by writing algebraic expressions. You wrote algebraic expressions for all four operations: addition, subtraction, multiplication, and division.

You learned that it helps to draw and label bar models to solve real-world problems involving variables. You also learned that you can substitute numbers into your expressions to replace the variables to see if your expressions are correct.

☑ **Complete Math Checkpoint**

7.5 Real-World Problems: Algebraic Expressions

Lesson Objective

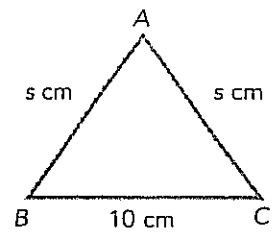
- Solve real-world problems involving algebraic expressions.

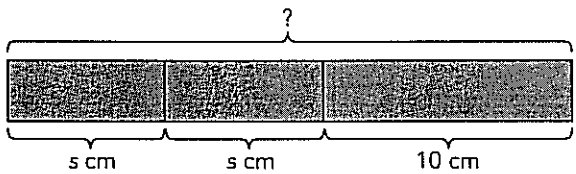
Learn

Write an addition or subtraction algebraic expression for a real-world problem and evaluate it.

The figure shows a triangle *ABC*.

a) What is the perimeter of the triangle *ABC* in terms of *s*?

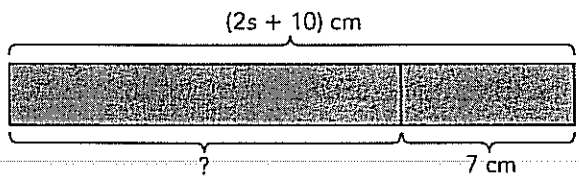




$s + s + 10 = 2s + 10$

The perimeter of the triangle *ABC* is $(2s + 10)$ centimeters.

b) The perimeter of a trapezoid is 7 cm shorter than the perimeter of triangle *ABC*. Find the perimeter of the trapezoid.



$2s + 10 - 7 = 2s + 3$

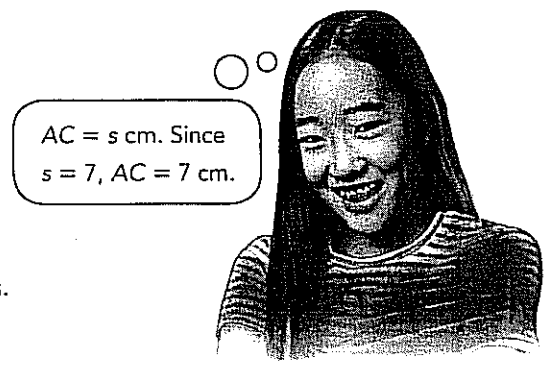
The perimeter of the trapezoid is $(2s + 3)$ centimeters.

c) If $s = 7$, find the perimeter of the triangle *ABC*.

When $s = 7$,

$$\begin{aligned} 2s + 10 &= (2 \cdot 7) + 10 \\ &= 14 + 10 \\ &= 24 \end{aligned}$$

The perimeter of the triangle *ABC* is 24 centimeters.



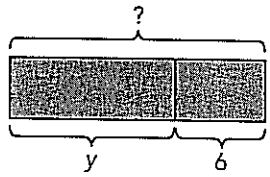
Guided Practice

Complete.

1 Raoul is y years old. Kayla is 6 years older than Raoul and Isaac is 4 years younger than Raoul.

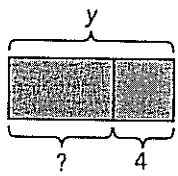
a) Find Kayla's age.

Kayla is ___?___ years old.



b) Find Isaac's age.

Isaac is ___?___ years old.



c) If $y = 12$, find the sum of Raoul's age and Isaac's age.

When $y = 12$,

Isaac's age:

___?___ − ___?___ = ___?___ − ___?___

= ___?___ years old

Sum of Raoul's age and Isaac's age:

___?___ + ___?___ = ___?___

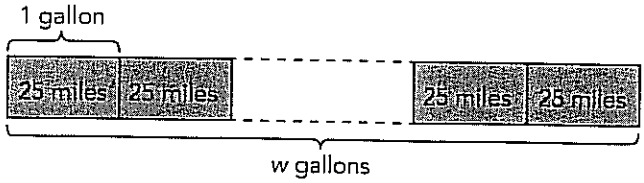
The sum of Raoul's age and Isaac's age is ___?___ years.

Learn

Write a multiplication or division algebraic expression for a real-world problem and evaluate it.

A car uses 1 gallon of gas for every 25 miles traveled.

a) How far can the car travel on w gallons of gas?



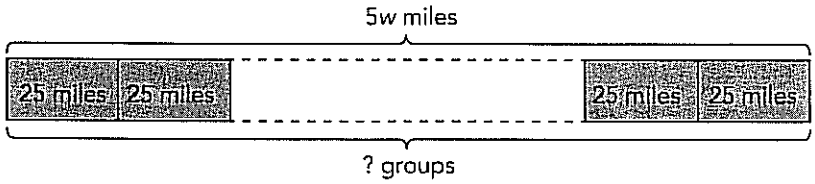
1 gallon ⟶ 25 miles

w gallons ⟶ $w \cdot 25 = 25w$ miles

The car can travel $25w$ miles on w gallons of gas.

Continue on next page

b) How much gas is used if the car travels $5w$ miles? Evaluate this expression when $w = 72$.



25 miles ⟶ 1 gallon

$5w$ miles ⟶ $5w \div 25 = \frac{5w}{25}$

$= \frac{w}{5}$ gallons

$\frac{w}{5}$ gallons of gas is used.

When $w = 72$,

$\frac{w}{5} = \frac{72}{25}$

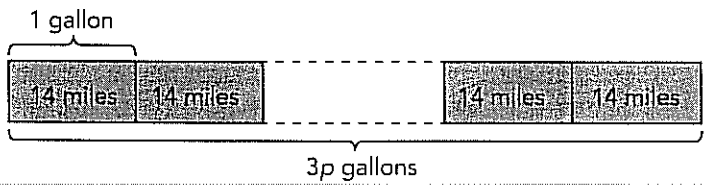
$= 14.4$

Guided Practice

Complete.

2 A pickup truck uses 1 gallon of gas for every 14 miles traveled.

a) How far can it travel on $3p$ gallons of gas?

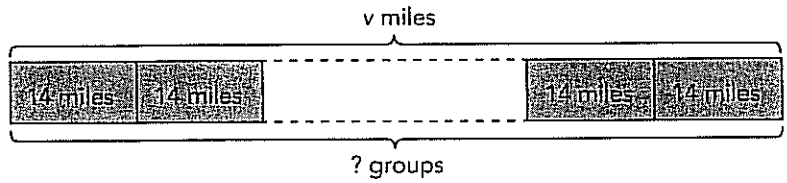


1 gallon ⟶ ___?___ miles

$3p$ gallons ⟶ ___?___ · ___?___ = ___?___ miles

It can travel ___?___ miles on $3p$ gallons of gas.

b) How many gallons of gas have been used after the pickup truck has traveled v miles? Evaluate this expression when $v = 56$.



14 miles → ___?___ gallon

v miles → ___?___ ÷ ___?___ = ___?___ gallons

___?___ gallons have been used.

When $v = 56$,

___?___ = ___?___

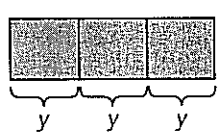
= ___?___

Learn

Write an algebraic expression using several operations and evaluate it.

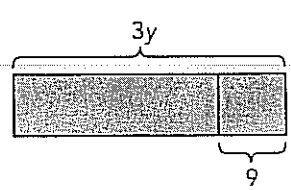
Gillian thinks of a number y. She multiplies it by 3 and then subtracts 9 from the product. What is her answer? Evaluate this expression when $y = 12$.

y multiplied by 3:



$y \cdot 3 = 3y$

Subtract 9 from the product:



$3y - 9$

Gillian's answer is $3y - 9$.

When $y = 12$,

$3y - 9 = (3 \cdot 12) - 9$

$= 36 - 9$

$= 25$

Practice 7.5

1 Jenny is x years old. Thomas is 3 times as old as she is. Jenny is 5 years older than Alexis.

a) Find Alexis's age in terms of x.

b) Find Thomas's age in terms of x.

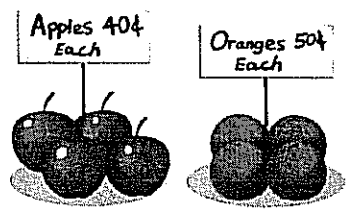
c) If $x = 12$, how much older is Thomas than Jenny?

2 A van travels from Town A to Town B. It uses 1 gallon of gas for every 24 miles traveled.

a) How many gallons of gas does the van use if it travels $3x$ miles?

b) The van uses $2y$ gallons of gas for its journey from Town A to Town B. Find the distance between Town A and Town B.

3 Brian bought x apples and some oranges. Brian bought 3 more oranges than apples.



a) Find the total number of fruit Brian bought in terms of x.

b) Find the total amount of money, in cents, that Brian spent on the fruit. Give your answer in terms of x.

c) If Brian could have bought exactly 12 pears with the amount of money that was spent on the apples and oranges, find the cost of each pear, in cents, in terms of x.

4 A rectangle has a width of x centimeters and a perimeter of $8x$ centimeters. A square has sides of length $\frac{1}{4}$ that of the length of the rectangle.

a) Find the length of the rectangle.

b) Find the perimeter of the square.

c) Find how many centimeters greater the rectangle's perimeter is than the square's perimeter if $x = 4$.

d) Find how many square centimeters greater the rectangle's area is than the square's area if $x = 4$.

Lesson 138

More Dot Plots

CCSS 6.SP.2d, 5
K–12.MP4

Objectives
- Identify the shape of a dot plot.
- Interpret a dot plot.

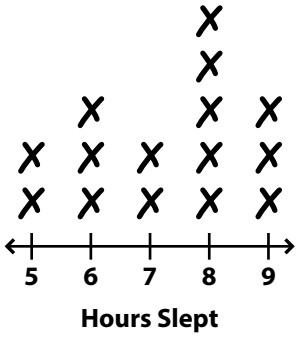
Books & Materials
- *Math in Focus* B
- 2 number cubes

Assignments
- ❑ Complete Warm-up.
- ❑ Read and complete pp. 223–225, *Math in Focus* B.
- ❑ Complete problems 3–8, pp. 226–227, *Math in Focus* B.
- ❑ Complete Math Checkpoint.

Warm-up

Use the line plot to answer the questions.

The line plot shows the number of hours Kim slept each day for the last 15 days.



1. How many days did Kim sleep 8 hours?
2. How many days did Kim sleep at least 7 hours?
3. How many more days did Kim sleep more than 7 hours than less than 7 hours?

Instruction

Read **Learn** on p. 223 in ***Math in Focus***. Unlike a figure that has symmetry, a dot plot that is considered *symmetrical* does not have to be exactly symmetrical. As long as most of the data is near the center, the dot plot is balanced, or symmetrical. A dot plot is *skewed* if many more dots occur on the left or right of the area on the number line with the greatest concentration of frequencies.

Read **Learn** on p. 224. While the value 20 is not in the exact middle of the line, it is the value represented most often, with a frequency of 4. Therefore, the dot plot can be described as symmetrical, with most occurrences centered around 20.

The term *range* means the same as it does when working with sets of whole numbers. To find the range of a dot plot, subtract the least value from the greatest value. The range has nothing to do with the frequency of a value, as long as that frequency is at least 1. For example, in problem **b**, the range is $8 - 3 = 5$, not $9 - 1 = 8$.

Complete **Guided Practice** on pp. 224–225. Look for whether the majority of the data is balanced, or if it is tilted to one side of the dot plot.

Complete **Hands-On Activity** on p. 225. A frequency table is helpful for collecting data because it takes up less space than a dot plot.

To the Learning Guide

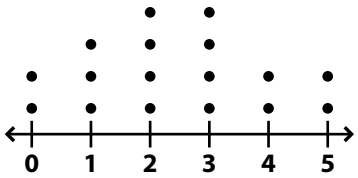
If your student is working alone, consider being a partner as he completes **Hands-On Activity**. Your student should notice that a frequency table and a dot plot have the same purpose. Either can be used to collect data, but the dot plot will additionally present the shape of the data.

Practice

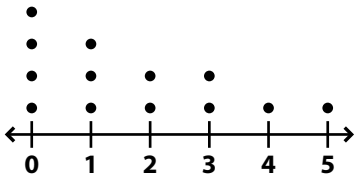
Complete problems 3–8 of **Practice 13.2** on pp. 226–227 in ***Math in Focus***. Then continue to work on your project.

Wrap-up

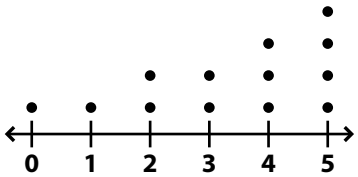
The shape of a dot plot is described as symmetrical if most of the data is near the center of the range.



The shape is described as skewed if most of the data in the distribution is either to the right or to the left. The following dot plot is right-skewed because it has a “tail” on the right. In other words, most (or all) occurrences appear to the right of 0, which is the value with the most occurrences.



The following dot plot is left-skewed because it has a "tail" on the left. This means that most (or all) occurrences appear to the left of 5, which is the value with the most occurrences.



The range of a dot plot is the difference between the least number with at least 1 dot and the greatest number with at least 1 dot.

☑ **Complete Math Checkpoint**

13.2 Dot Plots

Lesson Objective

- Display and analyze data using a dot plot.

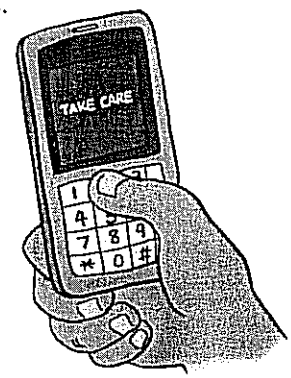
Vocabulary

dot plot	symmetrical
skewed	range

Learn

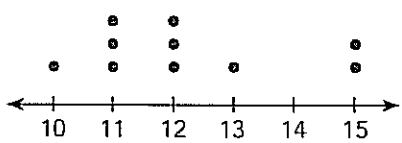
Represent numerical data using a dot plot.

a) The number of text messages sent by some students one day is as follows. Represent the data on a dot plot.



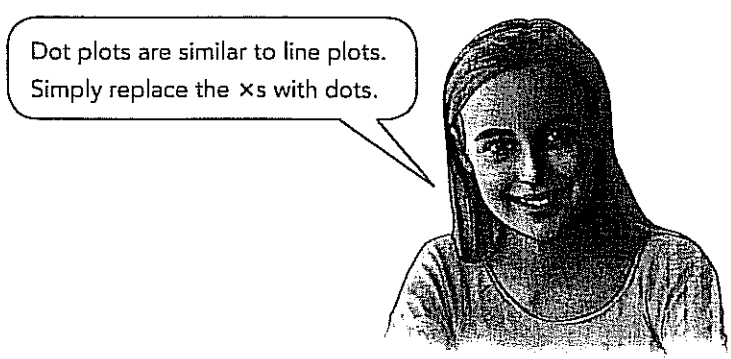
12	11	10	15	13
15	12	11	12	11

The number of text messages spans from 10 to 15 messages.
To construct a dot plot of these numbers, draw a horizontal number line that extends from 10 to 15.



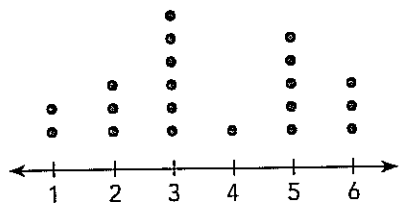
Number of Text Messages Sent

The data value 10 appears once, so place one dot above the number 10 on the number line. The data value 11 appears three times, so place three dots above the number 11.
The data value 14 is absent, so there is no dot above the number 14.



b) Eugenia tossed a number cube, numbered 1 to 6, 20 times. She recorded the number in the table below. Represent the data on a dot plot.

Number	1	2	3	4	5	6
Frequency	2	3	6	1	5	3



Numbers Tossed on a Number Cube

Guided Practice

Represent each set of data with a dot plot. Give each dot plot a title.

1. A group of 15 students was asked how many times they have traveled on a plane. The results are recorded in the table.

Number of Plane Trips	0	1	2	3	4	5	6
Number of Students	1	5	3	1	1	3	1

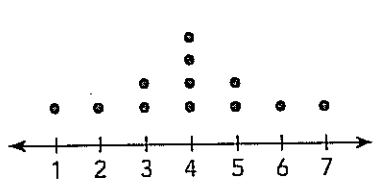
2. The results of the high jumps at a track meet are recorded in the table.

High Jump Results (cm)	150	151	152	153	154	155
Number of Students	1	4	3	1	0	2

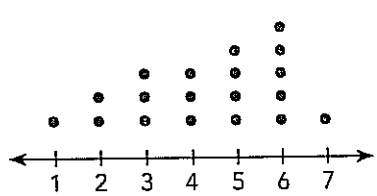
Learn

Identifying the shape of a set of data.

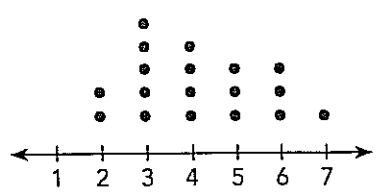
The shape of a dot plot can be either **symmetrical** or **skewed**.



Most of the data is near the center of the range. The shape is symmetrical.



The dot plot has a "tail" on the left. The shape is left-skewed.

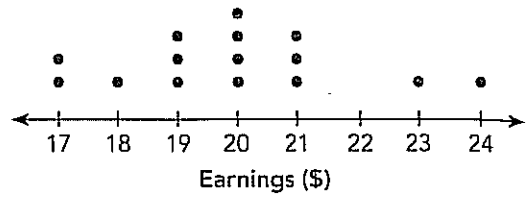


The dot plot has a "tail" on the right. The shape is right-skewed.

Learn

Interpret data from a dot plot.

a) The money (in dollars) that 15 students earned by babysitting in one week is shown in the dot plot on the right. Briefly describe the distribution of the earnings of the students.



A dot plot can be described by its shape and its **range**.

This dot plot is nearly symmetrical.

These data show a nearly symmetrical dot plot centered around 20. Most of the data values are from 19 to 21.

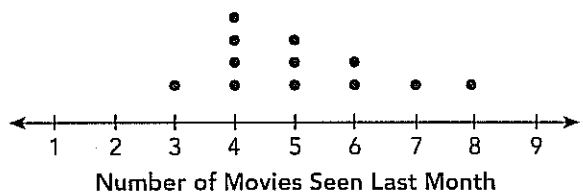
The data values are from 17 to 24.

The range of the data is the difference of these numbers.

Range: $24 - 17 = 7$

From these characteristics, you can describe the weekly earnings of the babysitters. They typically earn from \$19 to \$21 each week, but their earnings range from \$17 to \$24.

b) The number of movies that 12 students saw last month is shown in the dot plot on the right. Briefly describe the number of movies seen by the students.



The 12 dots represent 12 students. The dot plot has a "tail" on the right. Most of the data value are from 4 to 6, and the distribution is skewed to the right. The data values are from 3 to 8.

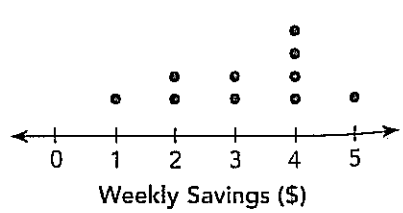
Range: $8 - 3 = 5$

From the description of the dot plot, you know that the students saw about 4 or 5 movies last month, and all of them saw 3 to 8 movies.

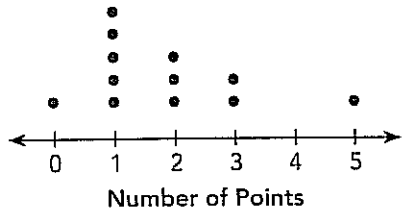
Guided Practice

Describe the data.

3 The weekly savings (in dollars) of 10 students in a class are shown in the dot plot. Briefly describe the distribution of the weekly savings of the students.



4 The number of points scored by 12 members of a volleyball team in a game is shown in the dot plot. Briefly describe the number of points scored by the group of players.



Hands-On Activity

Materials:

- 2 number cubes, numbered 1–6
- blank table

CONSTRUCTING A DISTRIBUTION

Work in pairs.

STEP **1** Toss two number cubes. Record the difference between the two numbers in a tally chart.

STEP **2** Repeat 20 times and add your results to another group's results. Record the results for 40 tosses in a copy of the table below.

Difference	0	1	2	3	4	5
Frequency	?	?	?	?	?	?

STEP **3** Represent the data with a dot plot.

STEP **4** Repeat STEP **1** to STEP **3**, but record the SUM of the two numbers each time.

a) What is the least sum you can get?

b) What is the greatest sum you can get?

Math Journal

a) Describe the distribution of the differences.

b) Describe the distribution of the sums.

c) Discuss with your partner why one distribution is skewed, and the other is symmetric (or nearly so). Why are the shapes of the two distributions different?

Practice 13.2

Represent each set of data with a dot plot.

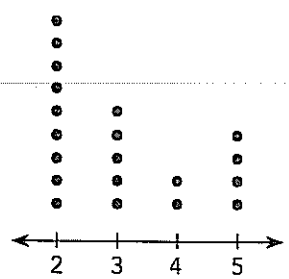
1. The years of service for each of the 18 employees in a company are as follows:

7	8	4	3	10	3
2	10	7	6	8	2
1	4	11	12	6	9

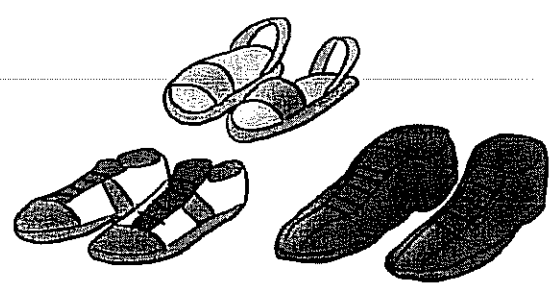
2. A group of 24 students was asked how many states they have visited. The results are recorded in the table.

Number of States Visited	0	1	2	3	4	5	6	7
Number of Students	2	4	5	2	2	6	2	1

A group of teens was asked to indicate how many pairs of shoes they have in their closet. The results are shown in the dot plot. Use the dot plot to answer questions 3 to 5.



Number of Pairs of Shoes

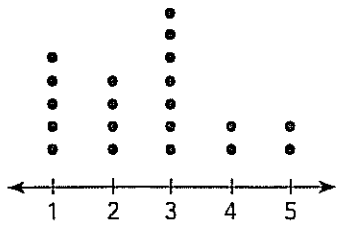


3. How many data values are there?

4. What conclusions can you draw with regard to the number of pairs of shoes the teens have?

5. What percent of the teens indicated 3 pairs of shoes in their closet?

The dot plot shows the number of weeks each movie that was number 1 at the box office during one year stayed in the number 1 position. Use the dot plot to answer questions 6 and 7.



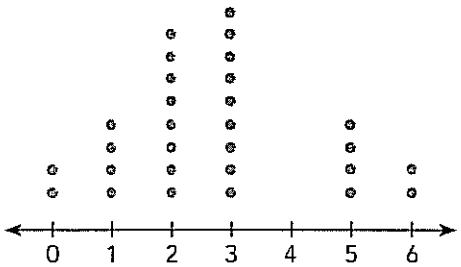
Number of Weeks as Number 1 Movie

6 How many movies are represented by the dots altogether?

7 Give a reason why more dots are above the numbers 1 to 3 than above the numbers 4 and 5 on the horizontal axis.

Copy and complete the dot plot. Use the dot plot to answer each question.

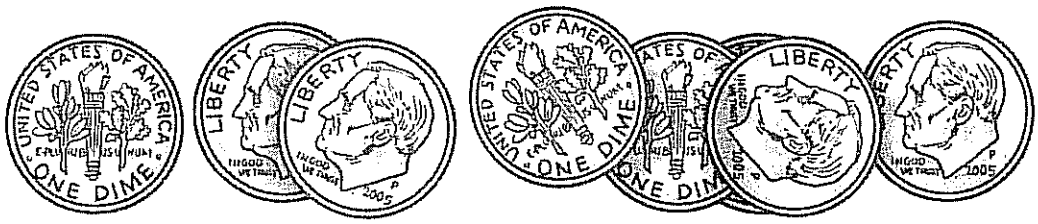
8 The incomplete dot plot shows the result of a survey in which each student was asked how many dimes were in their pockets or wallets. The results for "4 dimes" are not shown. Each dot represents one student. It is known that 12.5% of the students had one dime.



Number of Dimes

a) Find the number of students surveyed. Then complete the dot plot.

b) What percent of the students had either 0 or 6 dimes?

c) What percent of the students had either 1 or 5 dimes?

d) Briefly describe the distribution of the data.