About Singapore Math

This math course is based on the textbook series ***Math in Focus***, which was developed using the principles of the highly successful Singapore math program. Singapore math first drew international attention in 1999, when the first Trends in International Mathematics and Science Study (TIMMS) was conducted. In that year, and in the following testing years, students from Singapore led the world in math achievement. The ***Math in Focus*** series was written to bring the methods of Singapore math instruction to American students.

Why did we choose Singapore math?

There are several core principles that are important for you to know and to emphasize as you work with your student in this math course.

Fewer Topics; More Depth

If you skim the Table of Contents at the front of ***Math in Focus***, you will see that most of the chapters focus on the real number system, expressions and equations, and statistical analysis. By the end of this course, your student will have manipulated numbers in so many different ways that he will have a solid foundation in these concepts and will need little, if any, review in subsequent courses. Singapore math courses focus on just a few topics each year, which are taught in depth to assure mastery.

Start with the Concrete

Math in Focus contains many photographs of real objects. No matter what concept is being taught, Singapore math always begins with items that your student can experience and manipulate. Only after the concept has been well developed will he move to pictorial representations, and only after the pictorial is understood will he move to the abstract.

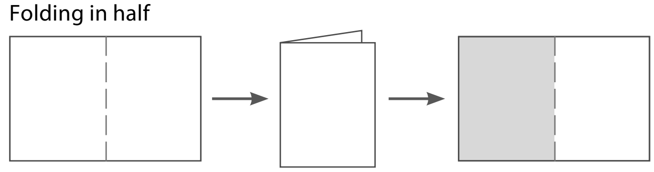
For example, when learning to divide fractions, your student will begin by dividing paper strips into different fractional amounts (concrete).

After this, your student will complete fraction-based word problems using pictures of shapes divided into different fractional amounts (pictorial).

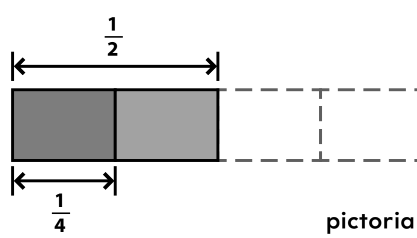
Finally, your student will be asked to divide different fractions (abstract).

For your student to grasp and master the mathematical concepts presented in this course, it is essential that you follow this progression as you are presenting new material to your student.

Folding in half



concrete



pictorial

$$\frac{1}{2} + \frac{1}{4} = \frac{?}{?}$$

abstract

07M-LM A

Mental Math

Singapore math emphasizes the importance of calculating quickly and accurately without the use of pencil and paper. Your student will learn and practice mental math strategies.

Solving Problems

According to Singapore math philosophy, the primary purpose for learning math is to learn to solve problems. Not only does math provide skills to solve problems involving calculation, but also strategies to solve problems your student might face in life: breaking a problem down into smaller parts, drawing a diagram, making a list, etc. Problems do take more time to solve, so do not skip or hurry through them.

You can learn more about teaching problem-solving skills in the brief *How to Teach Problem Solving* tutorial video in the Online Math Manual.

Bar Modeling

Another essential Singapore math tool is the bar model. Bar models are specialized drawings that are used to visualize a problem and organize the information. Your student can use them to see how to find a solution.

Go to the Online Math Manual to view the brief *How to Teach Bar Modeling* tutorial video.

Lesson 2

Squares, Cubes, and Their Roots

CCSS K–12.MP7

Objectives
- Find squares and cubes.
- Find square roots of perfect squares and cube roots of perfect cubes.

Books & Materials
- ***Math in Focus*** A

Assignments
- ❑ Complete Warm-up.
- ❑ Read and complete p. 5, ***Math in Focus*** A.
- ❑ Complete Math Checkpoint.

Warm-up

Solve.

1. $3 \cdot 3$
2. $3 \cdot 3 \cdot 3$
3. Use $<$ to compare 9 and 27.

Instruction

Today's lesson covers important ideas you need to understand before beginning the lessons in this chapter. Read p. 5 in ***Math in Focus***.

Recall that, to find the square of a number, you multiply the number by itself.

$$3^2 = 3 \cdot 3 = 9$$

Make sure you do not accidentally multiply the base by the exponent; 3^2 does **not** equal $3 \cdot 2$.

To find the cube of a number, multiply the number by itself three times.

$$3^3 = 3 \cdot 3 \cdot 3 = 27$$

To find a square root, think: What number times itself equals this number? You may find it helpful to guess and check.

To find the cube root of a number, find what number multiplied by itself three times equals the original number.

You can compare squares, cubes, square roots, and cube roots by first evaluating the expressions. For example, you can order $\sqrt[3]{27}$, 4^2, $\sqrt{4}$ and 4^3 by first writing this list of numbers as 3, 16, 2, and 64. Then you can order the numbers as $2 < 3 < 16 < 64$ and finally rewrite the numbers in their original form: $\sqrt{4} < \sqrt[3]{27} < 4^2 < 4^3$.

Helpful Online Resources

Instructional Video: *Square Roots*

To the Learning Guide

As you discuss these pages with your student, take note of the skills that seem familiar and those that may need review. Do not provide detailed instruction at this point; simply preview the material to prepare for the **Quick Check**.

Skills Check

Complete **Quick Check** on p. 5 in ***Math in Focus***.

To the Learning Guide

Review your student's answers to the **Quick Check**, noting the problems that your student answered incorrectly. Then go online through your student's portal to today's lesson. Click on the link to access the appropriate **Reteach** activity that your student should complete for the remainder of this lesson.

Wrap-up

Today you reviewed how to find squares and cubes.

$$5^2 = 5 \cdot 5$$
$$= 25$$
$$5^3 = 5 \cdot 5 \cdot 5$$
$$= 125$$

You reviewed how to find square roots and cube roots.

$$\sqrt{25} = 5$$
$$\sqrt[3]{125} = 5$$

You also reviewed how to order a list of numbers containing squares, cubes, square roots, and cube roots.

$$\sqrt[3]{64} < \sqrt{36} < 2^3 < 3^2$$

☑ **Complete Math Checkpoint**

Lesson 3

Absolute Value

CCSS 7.NS.1
K–12.MP6

Objectives
- Find the absolute value of rational numbers.

Books & Materials
- ***Math in Focus*** A
- tape or string *(Optional)*

Assignments
- ❑ Complete Warm-up.
- ❑ Read and complete pp. 6–9 (top), ***Math in Focus*** A.
- ❑ Complete problems 1–4 and 28–35, pp. 14–15, ***Math in Focus*** A.
- ❑ Complete Math Checkpoint.

Warm-up

Find each square, cube, or root.

1. 18^2
2. 9^3
3. $\sqrt{169}$
4. $\sqrt[3]{125}$

Instruction

Read p. 6 in ***Math in Focus***. The sign in front of a number indicates its location with respect to 0 on a number line. If a number has no sign, you can assume it is positive. Negative numbers are to the left of 0, and positive numbers are to the right. The *absolute value* of a number tells you how far the number is from zero. Since distances are always positive, the absolute value of any number is positive. What is the absolute value of 0? Since 0 is already at 0 on the number line, its absolute value is zero.

Complete **Quick Check** on p. 6.

Read **Find the Absolute Values of Positive Fractions** on p. 7. Just as with integers, the absolute value of a fraction is its distance from zero on a number line.

Then read p. 8. The *opposite* of an integer is the number on the opposite side and same distance away from zero on the number line. In the same way, the opposite of a fraction is the fraction on the opposite side and same distance away from zero on the number line. List two pairs of fractions that are opposites. One example is $\frac{1}{2}$ and $-\frac{1}{2}$.

Read **Example 1** on p. 9. Notice that the farther a number is from zero on the number line, the greater its absolute value. Even if one number is positive and one is negative, if the negative number is farther from zero, its absolute value is greater.

Complete **Guided Practice** on p. 9.

Helpful Online Resources

Instructional Video: *Integers, Rational Numbers, and Absolute Value*

BrainPOP: *Absolute Value*

To the Learning Guide

Watch For These Common Errors

! Some students confuse finding the absolute value of a number with finding the opposite of a number. For example, your student may say that the absolute value of $-\frac{2}{3}$ is $\frac{2}{3}$ and that the absolute value of $\frac{1}{5}$ is $-\frac{1}{5}$. He must understand that the absolute value of any number is positive.

Practice

Complete problems 1–4 and 28–35 of **Practice 1.1** on pp. 14–15 in ***Math in Focus***.

Wrap-up

Today you learned how to find the absolute value of fractions and decimals.

$$\left|\frac{4}{5}\right| = \frac{4}{5}$$

$$\left|-\frac{15}{6}\right| = \frac{15}{6}$$

$$|-2.7| = 2.7$$

You also learned to compare numbers' distances from zero.

-1.86 is closer to zero than $2\frac{6}{11}$ because $|-1.86| < \left|2\frac{6}{11}\right|$.

☑ **Complete Math Checkpoint**

Finding squares, cubes, square roots, and cube roots

Find the square and cube of 4. The square of a number is its second power and the cube of a number is its third power.

$$
\begin{aligned}
4^2 &= 4 \cdot 4 \\
&= 16
\end{aligned}
$$

$$
\begin{aligned}
4^3 &= 4 \cdot 4 \cdot 4 \\
&= 64
\end{aligned}
$$

The square of 4 is 16.

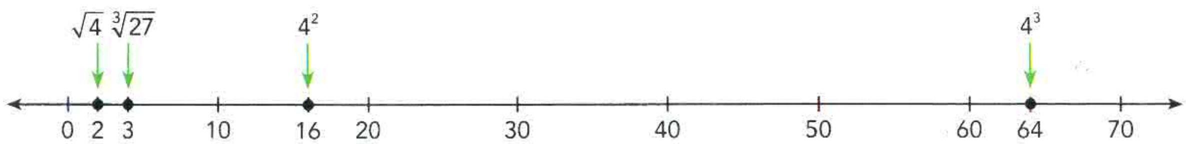
The cube of 4 is 64.

Find the square root of 4 and the cube root of 27.

$\sqrt{4} = 2$

$\sqrt[3]{27} = 3$

The square root of 4 is 2, because $2^2 = 4$.
The cube root of 27 is 3, because $3^3 = 27$.



On the number line, $\sqrt{4}$ is closest to 0 and 4^3 is farthest from 0. To describe these numbers in relation to one another, you may express them as follows:

$\sqrt{4} < \sqrt[3]{27} < 4^2 < 4^3$

Quick Check

Find the square of each number.

14 3

15 12

Find the cube of each number.

16 5

17 6

Find the square root and cube root of each number.

18 64

19 729

Order the numbers from greatest to least. Use the > symbol.

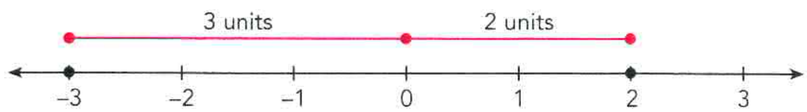
20 $\sqrt{81}$, 8^2, 3^3

Determining absolute values

The absolute value of a number n is denoted by $|n|$.

Examples: $|2| = 2$, $|-3| = 3$

The absolute value of a number is a measure of its distance from 0.



The distance from -3 to 0 is 3 units.

The distance from 2 to 0 is 2 units.

Quick Check

Use the following set of numbers for questions 21 to 25.

34, -23, -54, 54, -60

21 Find the absolute value of each number.

22 Which number is closest to 0?

23 Which number is farthest from 0?

24 Name two numbers with the same absolute value.

25 Which number has the greatest absolute value?

Use the number line to find the absolute value of each of the following numbers.

26 $|-15|$

27 $|6|$

28 $|-2.1|$

Copy and complete each ? with $>$, $=$, or $<$.

29 $|-7|$? $|-72|$

30 $|5|$? $|-5|$

31 $|-26|$? $|5|$

1.1 Representing Rational Numbers on the Number Line

Lesson Objectives

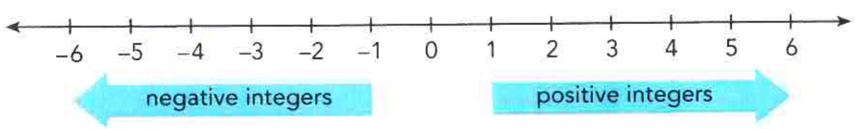
- Find the absolute values of rational numbers.
- Express numbers in $\frac{m}{n}$ form.
- Locate rational numbers on the number line.

Vocabulary

opposites	positive integers
set of integers	negative fractions
negative integers	rational numbers

Find the Absolute Values of Positive Fractions.

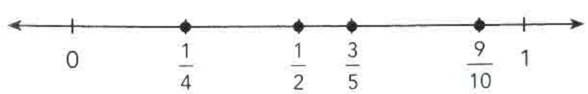
Previously, you learned how to graph whole numbers and negative numbers on a number line. The set of whole numbers and their **opposites** is called the **set of integers**.



The numbers on the right of 0 are called **positive integers**. The numbers on the left of 0 are called **negative integers**. The number 0 itself is neither positive nor negative.

There are gaps between the integers on the number line. These gaps contain fractions.

In the gap between 0 and 1, you can write proper fractions such as $\frac{1}{4}$, $\frac{1}{2}$, $\frac{3}{5}$, and $\frac{9}{10}$.



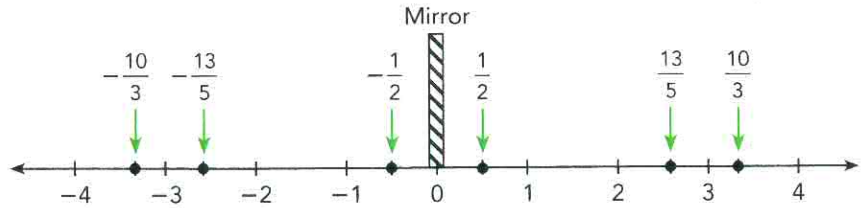
The integer 1 is one unit from 0. So, the fraction $\frac{1}{2}$ must be $\frac{1}{2}$ unit from 0. You can write this distance as $\left|\frac{1}{2}\right|$. The measure of the distance of other fractions from 0 is defined in the same way.

The absolute value of a positive fraction is just the fraction itself.

Examples: $\left|\frac{1}{4}\right| = \frac{1}{4}$, $\left|\frac{3}{5}\right| = \frac{3}{5}$, and $\left|2\frac{5}{8}\right| = \frac{21}{8}$, because $2\frac{5}{8} = \frac{21}{8}$.

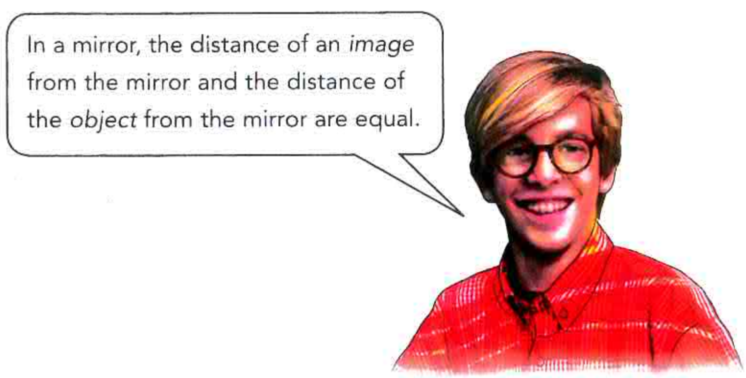
Find the Absolute Values of Negative Fractions and Decimals.

You might imagine that a mirror is placed on the number line at the number 0.



As you look into the mirror, you see the images of the positive integers. These images are the negative integers.

In the same way, fractions such as $\frac{1}{2}$, $\frac{13}{5}$, and $\frac{10}{3}$ each has an opposite in the mirror. The negative fractions are $-\frac{1}{2}$, $-\frac{13}{5}$, and $-\frac{10}{3}$.



The absolute value of a negative fraction is defined as the distance of the negative fraction from 0. You find the absolute value of the negative fractions as you do negative integers.

Examples:

$$\left|-\frac{10}{3}\right| = \frac{10}{3}$$

$$\left|-\frac{13}{5}\right| = \frac{13}{5}$$

$$|-1.35| = 1.35$$

So, the distance of $-\frac{10}{3}$ from 0 is $\frac{10}{3}$ units. In the same way, $-\frac{13}{5}$ is $\frac{13}{5}$ units from 0 and -1.35 is 1.35 units from 0.

Example 1 Find the absolute values of fractions.

Solve. Show your work.

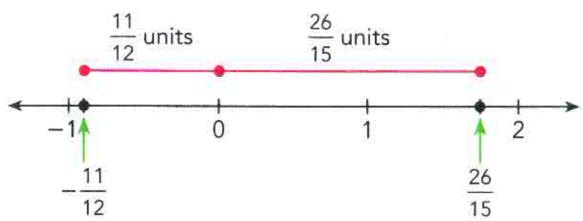
a) Find the absolute values of $-\frac{11}{12}$ and $\frac{26}{15}$.

Solution

$\left|-\frac{11}{12}\right| = \frac{11}{12}$ and $\left|\frac{26}{15}\right| = \frac{26}{15}$.

b) Using a number line, show how far $-\frac{11}{12}$ and $\frac{26}{15}$ are from 0. Which number is closer to 0?

Solution



$-\frac{11}{12}$ is $\frac{11}{12}$ units to the left of 0. $\frac{26}{15}$ is $\frac{26}{15}$ units to the right of 0.

Because the distance $\frac{11}{12} < 1$ unit and $\frac{26}{15} > 1$ unit, $-\frac{11}{12}$ is closer to 0.

Guided Practice

Solve.

1. Find the absolute values of $3\frac{2}{7}$ and $-\frac{18}{5}$.

2. Graph the two numbers on a number line and indicate their distances from 0. Which number is farther from 0?

Express Integers and Fractions in $\frac{m}{n}$ Form.

A **rational number** is a number which can be written as $\frac{m}{n}$ where m and n are integers with $n \neq 0$. The definition of rational numbers comes from the concept of fractions.

Examples: $1\frac{1}{2} = \frac{3}{2}$. So, $1\frac{1}{2}$ is a rational number.

$3 = \frac{3}{1}$. So, 3 is a rational number.

Think Math

You cannot write a rational number $\frac{m}{n}$ with $n = 0$. Why?

To express 0 in the form $\frac{m}{n}$, you can write 0 as $\frac{0}{1}$.

Example 2 Express integers and fractions in $\frac{m}{n}$ form.

Write each number in $\frac{m}{n}$ form where m and n are integers.

a) 23

b) -45

c) $11\frac{7}{9}$

d) $-\frac{16}{21}$

Solution

a) $23 = \frac{23}{1}$ — Whole numbers have 1 in the denominator.

b) $-45 = \frac{-45}{1}$ — Negative integers have 1 in the denominator.

c) $11\frac{7}{9} = \frac{11 \cdot 9}{9} + \frac{7}{9}$ — Write $11\frac{7}{9}$ as an improper fraction.

$= \frac{106}{9}$

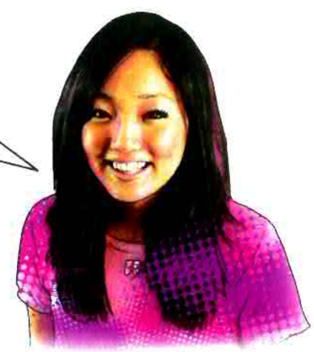
d) $-\frac{16}{21} = \frac{-16}{21}$ — Write the negative integer in either the numerator or denominator.

or $= \frac{16}{-21}$

For negative fractions, the negative integer may be placed in either the numerator or the denominator.

Examples: $-\frac{2}{9} = \frac{-2}{9} = \frac{2}{-9}$

$-3 = \frac{-3}{1} = \frac{3}{-1}$



Guided Practice

Write each number in $\frac{m}{n}$ form where m and n are integers.

3 $11\frac{1}{6}$

4 48

5 $-5\frac{4}{12}$

6 $-\frac{25}{10}$

Express Decimals in $\frac{m}{n}$ Form.

You have learned how to express decimals as fractions. Decimals also have their negative counterparts on the number line. So, you can write decimals in $\frac{m}{n}$ form.

Example 3 **Express decimals in $\frac{m}{n}$ form.**

Write each decimal as $\frac{m}{n}$ where m and n are integers with $n \neq 0$.

a) 0.4

b) −0.186

c) 30.5

d) −1.48

Solution

a) $0.4 = \frac{4}{10}$ — 4 is in the tenths place. Use 10 as the denominator.

$= \frac{2}{5}$ — Simplify.

b) $-0.186 = -\frac{186}{1{,}000}$ — 6 is in the thousandths place. Use 1,000 as the denominator.

$= \frac{-93}{500}$ — Simplify.

c) $30.5 = 30\frac{1}{2}$ — Write the integer, 30. Write 0.5 as $\frac{1}{2}$.

$= \frac{61}{2}$ — Write as an improper fraction.

d) $-1.48 = -1\frac{48}{100}$ — Write the integer, −1. 8 is in the hundredths place. Use 100 as the denominator.

$= \frac{-148}{100}$ — Write as an improper fraction.

$= \frac{-37}{25}$ — Simplify.

Guided Practice

Write each decimal as $\frac{m}{n}$ where m and n are integers with $n \neq 0$.

7. 11.5

8. −7.8

9. 0.36

10. −0.125

Locate Rational Numbers on the Number Line.

Rational numbers can be located on the number line easily.

Example 4 Locate rational numbers on the number line.

Locate the rational numbers $\frac{3}{5}$ and -2.4 on the number line.

Solution

STEP 1 Find the integers that the rational number lies between.

$\frac{3}{5}$ is a proper fraction so it is located between 0 and 1.

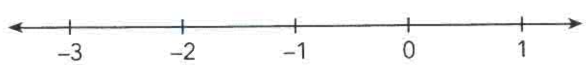
-2.4 is located between -3 and -2.

Caution

-2.4 can be written as a mixed number $-2\frac{2}{5}$ or $-2\frac{4}{10}$. Remember that it is a negative mixed number.

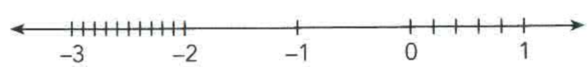
Make sure that you do not graph -2.4 by counting to the right of 0.

STEP 2 Graph a number line and label the integers.

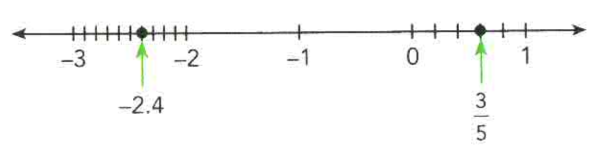


STEP 3 Divide the distance between the integers into equal segments.

You divide the distance between 0 and 1 into 5 equal segments and the distance between -3 and -2 into 10 equal segments.



STEP 4 Use the segments to locate $\frac{3}{5}$ and -2.4.



Guided Practice

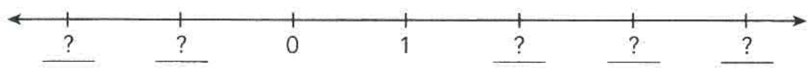
Copy and complete.

11 Locate the rational numbers -1.5 and $\frac{15}{4}$ on the number line.

STEP 1 Find the integers that the rational number lies between.

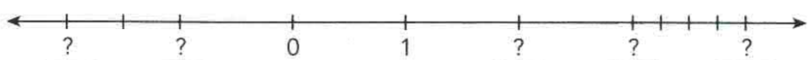
$\frac{15}{4}$ can be written as a mixed number, $3\frac{3}{4}$, and $3\frac{3}{4}$ lies between 3 and 4.
The negative decimal -1.5 lies between -2 and -1.

STEP 2 Graph a number line and label the integers.

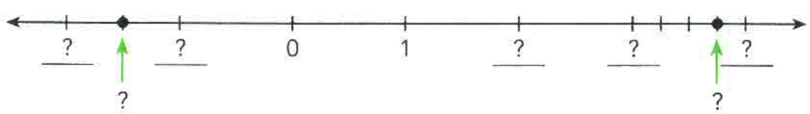


STEP 3 Divide the distance between the integers into equal segments.

You divide the distance between -2 and -1 into __?__ segments and the distance between 3 and 4 into __?__ segments.



STEP 4 Use the segments to locate -1.5 and $3\frac{3}{4}$.



Locate the following rational numbers on the number line.

12 $\frac{1}{6}$ and $\frac{15}{3}$

13 -0.4 and $\frac{11}{5}$

14 $\frac{12}{15}$ and -1.8

15 $-\frac{5}{15}$ and $-\frac{25}{30}$

Practice 1.1

Find the absolute value of each fraction. Use a number line to show how far the fraction is from 0. Write fractions in simplest form.

1 $\frac{7}{10}$

2 $\frac{18}{8}$

3 $-\frac{5}{13}$

4 $-\frac{48}{15}$

Write each integer or fraction as $\frac{m}{n}$ in simplest form where m and n are integers.

5 67

6 −345

7 $\frac{25}{80}$

8 $-\frac{264}{90}$

9 $-\frac{14}{70}$

10 $\frac{600}{480}$

Write each mixed number or decimal as $\frac{m}{n}$ in simplest form where m and n are integers.

11 $7\frac{7}{9}$

12 $-5\frac{1}{10}$

13 $2\frac{5}{12}$

14 $-10\frac{11}{36}$

15 0.4

16 −0.625

17 5.80

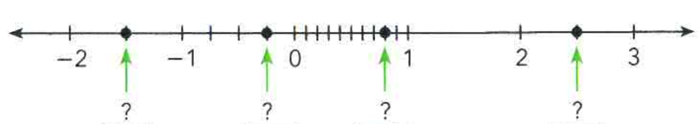
18 9.001

19 −10.68

Copy and complete.

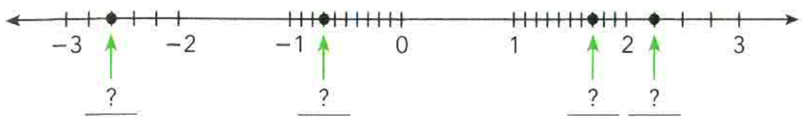
20 Locate the following rational numbers correctly on the number line.

$-\frac{1}{4}$, −1.5, 0.8, $\frac{5}{2}$



21 Locate the following rational numbers correctly on the number line.

$1\frac{7}{10}$, $-\frac{13}{5}$, 2.25, -0.7



Graph each rational number on a separate number line.

22 $67\frac{1}{8}$

23 $\frac{305}{20}$

24 $\frac{98}{28}$

25 $-\frac{21}{12}$

26 -25.8

27 -45.3

A video game gives you 10 minutes to find a treasure. The results of your first 8 games show the amount of time left unused when you have found the treasure. A negative time means you have gone beyond the 10 minutes allotted. Use these data for questions 28 to 35.

$\frac{23}{8}$, 0, $-7\frac{1}{5}$, 6, $-\frac{17}{4}$, 8, 7.8, -9.1

28 Order the times left from most to least time using the symbol >.

29 Write the absolute value of each number.

30 Which number has the greatest absolute value?

31 Order the absolute values from least to greatest. Use the symbol <.

32 Graph the original numbers on a number line.

33 Which negative number in the list is farthest from 0?

34 Which positive number in the list is closest to 10?

35 Which time is closest to -5 minutes?